Market And Product Diversification And Cost of Capital Implications

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When using Total Beta to estimate private company cost of capital we find a public company that is comparable to our private company, estimate the volatility of returns for that public company, and use that volatility estimate as a proxy for the return volatility of our private company. Is the volatility of that public company's returns a good proxy for the volatility of our private company's returns? If the public company is diversified by market and by product does this understate the volatility to be applied to the private company which may have only one product in only one market? In this paper we will use a one factor model to calculate the volatility of returns for a public company that has multiple products in multiple markets and then compare that volatility estimate to a comparable private company that has one product in one market.

Our Hypothetical Problem

Assume that ABC Company is a public company that manufactures and sells computer chips and fax machines in the United States and Europe. The table below presents ABC Company's expected annual returns and return volatilities by market and by product...

Business Line	Weight	Mean	Volatility
US chip business	40%	18%	30%
US fax machine business	30%	15%	25%
European chip business	20%	12%	20%
European fax machine business	10%	9%	15%
Total or weighted average	100%	15%	25%

It is a given that the economies of both the United States and Europe are correlated with the Global economy. It also follows that the chip and fax machine businesses in the United States are correlated with the state of the US economy and the chip and fax machine businesses in Europe are correlated with the state of the European economy. The table below presents ABC Company's relevant market and product correlations...

Symbol	Market or Product		Correlation
$\rho_{u,g}$	Correlation of the US market with the Global market	=	0.75
$\rho_{e,g}$	Correlation of the European market with the Global market	=	0.65
$\rho_{c,u}$	Correlation of the US chip business with the US market	=	0.85
$\rho_{f,u}$	Correlation of the US fax machine business with the US market	=	0.70
$\rho_{c,e}$	Correlation of the European chip business with the European market	=	0.80
$\rho_{f,e}$	Correlation of the European fax machine business with the European market	=	0.60
	Arithmetic average correlation	=	0.73

Note: The correlations above are hypothetical and do not necessarily represent real-world correlations although they are within the realm of reason.

Question: If we view ABC Company as a portfolio of business lines what is ABC Company's expected return and expected return volatility? By how much is ABC Company's portfolio volatility reduced as the result of diversification as compared to a company with one product in one market?

Market Factors

We will define **Market** as the continent or country in which ABC Company sells its products. ABC Company's two markets are the United States and Europe. We will model the correlation between markets via the common factor $\mathbf{M_G}$ (Global Market Factor). By definition M_G is a normally-distributed random variate with mean zero and variance one. Using this definition of M_G we get the following expectations...

$$M_G \sim N[0,1] \;\; ...such\,that... \;\; \mathbb{E}\Bigg[M_G\Bigg] = 0 \;\; ...and... \;\; \mathbb{E}\Bigg[M_G^2\Bigg] = 1 \tag{1}$$

The state of the economy in the United States is represented by the common factor $\mathbf{M_u}$ (US Market Factor). The value of M_u is a function of (1) the correlation between the US economy and the Global economy and (2) random noise, which is represented by the independent random variate Z_u. The correlation between the US economy and the Global economy is represented by $\rho_{u,g}$. The equation for the random variate M_u is...

$$M_u = \rho_{u,g}\, M_G + \sqrt{1-\rho_{u,g}^2}\, Z_u \;\; ...where... \;\; Z_u \sim N[0,1] \tag{2}$$

The expected values of M_u and the square of M_u (see Appendix equations (53) and (54)) are...

$$\mathbb{E}\Bigg[M_u\Bigg] = 0 \;\; ...and... \;\; \mathbb{E}\Bigg[M_u^2\Bigg] = 1 \tag{3}$$

Using the expectations above the mean (μ_u) and variance (σ_u^2) of M_u are...

$$\mu_u = \mathbb{E}\Bigg[M_u\Bigg] = 0 \tag{4}$$

$$\sigma_u^2 = \mathbb{E}\Bigg[M_u^2\Bigg] - \Bigg\{\mathbb{E}\Bigg[M_u\Bigg]\Bigg\}^2 = 1 \tag{5}$$

The state of the economy in Europe is represented by the common factor $\mathbf{M_e}$ (European Market Factor). The value of M_e is a function of (1) the correlation between the European economy and the Global economy and (2) random noise, which is represented by the independent random variate Z_e. The correlation between the European economy and the Global economy is represented by $\rho_{e,g}$. The equation for the random variate M_e is...

$$M_e = \rho_{e,g}\, M_G + \sqrt{1-\rho_{e,g}^2}\, Z_e \;\; ...where... \;\; Z_e \sim N[0,1] \tag{6}$$

The expected values of M_e and the square of M_e (see Appendix equations (55) and (56)) are...

$$\mathbb{E}\Bigg[M_e\Bigg] = 0 \;\; ...and... \;\; \mathbb{E}\Bigg[M_e^2\Bigg] = 1 \tag{7}$$

Using the expectations above the mean (μ_e) and variance (σ_e^2) of M_e are...

$$\mu_e = \mathbb{E}\Bigg[M_e\Bigg] = 0 \tag{8}$$

$$\sigma_e^2 = \mathbb{E}\Bigg[M_e^2\Bigg] - \Bigg\{\mathbb{E}\Bigg[M_e\Bigg]\Bigg\}^2 = 1 \tag{9}$$

The correlation between the US economy and the European economy in terms of the common factors M_u and M_e (see equations (5), (9), (53), (55) and (57)) is...

$$\rho_{u,e} = \frac{\mathbb{E}[M_u\, M_e] - \mathbb{E}[M_u]\mathbb{E}[M_e]}{\sqrt{\sigma_u^2}\sqrt{\sigma_e^2}} = \mathbb{E}\Bigg[M_u\, M_e\Bigg] = \rho_{u,g}\,\rho_{e,g} \tag{10}$$

For the hypothetical problem above the correlation between the US and European economies using Equation (10) is...

$$\rho_{u,e} = 0.75 \times 0.65 = 0.4875 \tag{11}$$

Product Factors

We will define **Product** as the products that ABC Company sells in it's respective markets. ABC Company's two products are computer chips and fax machines. We will model the correlation between the product and the market in which that product is sold via common factors (Product Factors) each of which is correlated with the United States economy if that product is sold in the United States and the European economy if that product is sold in Europe. The four product factors are...

Symbol		Description
C_u	=	Product factor for the chip business in the United States
F_u	=	Product factor for the fax machine business in the United States
C_e	=	Product factor for the chip business in Europe
F_e	=	Product factor for the fax machine business in Europe

The US product factors C_u and F_u are correlated with the US market factor M_u as defined by Equation (2) above. The correlation between C_u and M_u is represented by $\rho_{c,u}$ and the correlation between F_u and M_u is represented by $\rho_{f,u}$. The values of C_u and F_u are a function of (1) the correlation between the respective products and the US economy (M_u) and (2) random noise, which is represented by the independent random variates $Z_{c,u}$ and $Z_{f,u}$. The equations for the random variates C_u and F_u are...

$$C_u = \rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u} \ \ \textit{...where...} \ \ Z_{c,u} \sim N[0,1] \tag{12}$$

$$F_u = \rho_{f,u}\, M_u + \sqrt{1-\rho_{f,u}^2}\, Z_{f,u} \ \ \textit{...where...} \ \ Z_{f,u} \sim N[0,1] \tag{13}$$

The expected values of C_u, the square of C_u, F_u and the square of F_u (see Appendix equations (58), (59), (60) and (61)) are...

$$\mathbb{E}\Bigg[C_u\Bigg] = 0 \ \ \textit{...and...} \ \ \mathbb{E}\Bigg[C_u^2\Bigg] = 1 \tag{14}$$

$$\mathbb{E}\Bigg[F_u\Bigg] = 0 \ \ \textit{...and...} \ \ \mathbb{E}\Bigg[F_u^2\Bigg] = 1 \tag{15}$$

Using the expectations above the mean ($\mu_{c,u}$) and variance ($\sigma_{c,u}^2$) of C_u are...

$$\mu_{c,u} = \mathbb{E}\Bigg[C_u\Bigg] = 0 \tag{16}$$

$$\sigma_{c,u}^2 = \mathbb{E}\Bigg[C_u^2\Bigg] - \Bigg\{\mathbb{E}\Bigg[C_u\Bigg]\Bigg\}^2 = 1 \tag{17}$$

Using the expectations above the mean ($\mu_{f,u}$) and variance ($\sigma_{f,u}^2$) of F_u are...

$$\mu_{f,u} = \mathbb{E}\Bigg[F_u\Bigg] = 0 \tag{18}$$

$$\sigma_{f,u}^2 = \mathbb{E}\Bigg[F_u^2\Bigg] - \Bigg\{\mathbb{E}\Bigg[F_u\Bigg]\Bigg\}^2 = 1 \tag{19}$$

The European product factors C_e and F_e are correlated with the European market factor M_e as defined by Equation (6) above. The correlation between C_e and M_e is represented by $\rho_{c,e}$ and the correlation between F_e and M_e is represented by $\rho_{f,e}$. The values of C_e and F_e are a function of (1) the correlation between the respective products and the European economy (M_e) and (2) random noise, which is represented by the independent random variates $Z_{c,e}$ and $Z_{f,e}$. The equations for the random variates C_e and F_e are...

$$C_e = \rho_{c,e}\, M_e + \sqrt{1-\rho_{c,e}^2}\, Z_{c,e} \ \ \textit{...where...} \ \ Z_{c,e} \sim N[0,1] \tag{20}$$

$$F_e = \rho_{f,e}\, M_e + \sqrt{1-\rho_{f,e}^2}\, Z_{f,e} \ \ \textit{...where...} \ \ Z_{f,e} \sim N[0,1] \tag{21}$$

The expected values of C_e, the square of C_e, F_e and the square of F_e (see Appendix equations (62), (63), (64) and (65)) are...

$$\mathbb{E}\left[C_e\right] = 0 \;\; ...and... \;\; \mathbb{E}\left[C_e^2\right] = 1 \tag{22}$$

$$\mathbb{E}\left[F_e\right] = 0 \;\; ...and... \;\; \mathbb{E}\left[F_e^2\right] = 1 \tag{23}$$

Using the expectations above the mean ($\mu_{c,e}$) and variance ($\sigma^2_{c,e}$) of C_e are...

$$\mu_{c,e} = \mathbb{E}\left[C_e\right] = 0 \tag{24}$$

$$\sigma^2_{c,e} = \mathbb{E}\left[C_e^2\right] - \left\{\mathbb{E}\left[C_e\right]\right\}^2 = 1 \tag{25}$$

Using the expectations above the mean ($\mu_{f,e}$) and variance ($\sigma^2_{f,e}$) of F_e are...

$$\mu_{f,e} = \mathbb{E}\left[F_e\right] = 0 \tag{26}$$

$$\sigma^2_{f,e} = \mathbb{E}\left[F_e^2\right] - \left\{\mathbb{E}\left[F_e\right]\right\}^2 = 1 \tag{27}$$

Product Returns, Variances and Covariances

The equations for the annual returns on the chip ($r_{c,u}$) and fax machine ($r_{f,u}$) businesses in the United States are...

$$r_{c,u} = \mu_{c,u} + \sigma_{c,u}\, C_u \tag{28}$$

$$r_{f,u} = \mu_{f,u} + \sigma_{f,u}\, F_u \tag{29}$$

The expected return on the chip business in the United States (see Appendix equation (72)) is...

$$\begin{aligned} Mean(r_{c,u}) &= \mathbb{E}\left[r_{c,u}\right] \\ &= \mu_{c,u} \end{aligned} \tag{30}$$

The expected return variance on the chip business in the United States (see Appendix equations (72) and (76)) is...

$$\begin{aligned} Var(r_{c,u}) &= \mathbb{E}\left[r^2_{c,u}\right] - \left\{\mathbb{E}\left[r_{c,u}\right]\right\}^2 \\ &= \mu^2_{c,u} + \sigma^2_{c,u} - \mu^2_{c,u} \\ &= \sigma^2_{c,u} \end{aligned} \tag{31}$$

The expected return on the fax machine business in the United States (see Appendix equation (73)) is...

$$\begin{aligned} Mean(r_{f,u}) &= \mathbb{E}\left[r_{f,u}\right] \\ &= \mu_{f,u} \end{aligned} \tag{32}$$

The expected return variance on the fax machine business in the United States (see Appendix equations (73) and (77)) is...

$$\begin{aligned} Var(r_{f,u}) &= \mathbb{E}\left[r^2_{f,u}\right] - \left\{\mathbb{E}\left[r_{f,u}\right]\right\}^2 \\ &= \mu^2_{f,u} + \sigma^2_{f,u} - \mu^2_{f,u} \\ &= \sigma^2_{f,u} \end{aligned} \tag{33}$$

The equations for the annual returns on the chip ($r_{c,e}$) and fax machine ($r_{f,e}$) businesses in Europe are...

$$r_{c,e} = \mu_{c,e} + \sigma_{c,e} \, C_e \tag{34}$$

$$r_{f,e} = \mu_{f,e} + \sigma_{f,e} \, F_e \tag{35}$$

The expected return on the chip business in the Europe (see Appendix equation (74)) is...

$$\begin{aligned} Mean(r_{c,e}) &= \mathbb{E}\Bigg[r_{c,e}\Bigg] \\ &= \mu_{c,e} \end{aligned} \tag{36}$$

The expected return variance on the chip business in Europe (see Appendix equations (74) and (78)) is...

$$\begin{aligned} Var(r_{c,e}) &= \mathbb{E}\Bigg[r^2_{c,e}\Bigg] - \Bigg\{\mathbb{E}\Bigg[r_{c,e}\Bigg]\Bigg\}^2 \\ &= \mu^2_{c,e} + \sigma^2_{c,e} - \mu^2_{c,e} \\ &= \sigma^2_{c,e} \end{aligned} \tag{37}$$

The expected return on the fax machine business in the Europe (see Appendix equation (75)) is...

$$\begin{aligned} Mean(r_{f,e}) &= \mathbb{E}\Bigg[r_{f,e}\Bigg] \\ &= \mu_{f,e} \end{aligned} \tag{38}$$

The expected return variance on the fax machine business in Europe (see Appendix equations (75) and (79)) is...

$$\begin{aligned} Var(r_{f,e}) &= \mathbb{E}\Bigg[r^2_{f,e}\Bigg] - \Bigg\{\mathbb{E}\Bigg[r_{f,e}\Bigg]\Bigg\}^2 \\ &= \mu^2_{f,e} + \sigma^2_{f,e} - \mu^2_{f,e} \\ &= \sigma^2_{f,e} \end{aligned} \tag{39}$$

The covariance between the returns on the chip business in the United States and the returns on the fax machine business in the United States (see Appendix equations (72), (73) and (80)) is...

$$\begin{aligned} Cov(r_{c,u} r_{f,u}) &= \mathbb{E}\Bigg[r_{c,u} \, r_{f,u}\Bigg] - \mathbb{E}\Bigg[r_{c,u}\Bigg]\mathbb{E}\Bigg[r_{f,u}\Bigg] \\ &= \mu_{c,u} \, \mu_{f,u} + \rho_{c,u} \, \rho_{f,u} \, \sigma_{c,u} \, \sigma_{f,u} - \mu_{c,u} \, \mu_{f,u} \\ &= \rho_{c,u} \, \rho_{f,u} \, \sigma_{c,u} \, \sigma_{f,u} \end{aligned} \tag{40}$$

The covariance between the returns on the chip business in the United States and the returns on the chip business in Europe (see Appendix equations (72), (74) and (81)) is...

$$\begin{aligned} Cov(r_{c,u} r_{c,e}) &= \mathbb{E}\Bigg[r_{c,u} \, r_{c,e}\Bigg] - \mathbb{E}\Bigg[r_{c,u}\Bigg]\mathbb{E}\Bigg[r_{c,e}\Bigg] \\ &= \mu_{c,u} \, \mu_{c,e} + \rho_{u,m} \, \rho_{e,m} \, \rho_{c,u} \, \rho_{c,e} \, \sigma_{c,u} \, \sigma_{c,e} - \mu_{c,u} \, \mu_{c,e} \\ &= \rho_{u,m} \, \rho_{e,m} \, \rho_{c,u} \, \rho_{c,e} \, \sigma_{c,u} \, \sigma_{c,e} \end{aligned} \tag{41}$$

The covariance between the returns on the chip business in the United States and the returns on the fax machine business in Europe (see Appendix equations (72), (75) and (82)) is...

$$\begin{aligned} Cov(r_{c,u} r_{f,e}) &= \mathbb{E}\Bigg[r_{c,u} \, r_{f,e}\Bigg] - \mathbb{E}\Bigg[r_{c,u}\Bigg]\mathbb{E}\Bigg[r_{f,e}\Bigg] \\ &= \mu_{c,u} \, \mu_{f,e} + \rho_{u,m} \, \rho_{e,m} \, \rho_{c,u} \, \rho_{f,e} \, \sigma_{c,u} \, \sigma_{f,e} - \mu_{c,u} \, \mu_{f,e} \\ &= \rho_{u,m} \, \rho_{e,m} \, \rho_{c,u} \, \rho_{f,e} \, \sigma_{c,u} \, \sigma_{f,e} \end{aligned} \tag{42}$$

The covariance between the returns on the fax machine business in the United States and the returns on the chip business in Europe (see Appendix equations (73), (74) and (83)) is...

$$\begin{aligned} Cov(r_{f,u} r_{c,e}) &= \mathbb{E}\Big[r_{f,u}\, r_{c,e}\Big] - \mathbb{E}\Big[r_{f,u}\Big]\mathbb{E}\Big[r_{c,e}\Big] \\ &= \mu_{f,u}\,\mu_{c,e} + \rho_{u,m}\,\rho_{e,m}\,\rho_{f,u}\,\rho_{c,e}\,\sigma_{f,u}\,\sigma_{c,e} - \mu_{f,u}\,\mu_{c,e} \\ &= \rho_{u,m}\,\rho_{e,m}\,\rho_{f,u}\,\rho_{c,e}\,\sigma_{f,u}\,\sigma_{c,e} \end{aligned} \tag{43}$$

The covariance between the returns on the fax machine business in the United States and the returns on the fax machine business in Europe (see Appendix equations (73), (75) and (84)) is...

$$\begin{aligned} Cov(r_{f,u} r_{f,e}) &= \mathbb{E}\Big[r_{f,u}\, r_{f,e}\Big] - \mathbb{E}\Big[r_{f,u}\Big]\mathbb{E}\Big[r_{f,e}\Big] \\ &= \mu_{f,u}\,\mu_{f,e} + \rho_{u,m}\,\rho_{e,m}\,\rho_{f,u}\,\rho_{f,e}\,\sigma_{f,u}\,\sigma_{f,e} - \mu_{f,u}\,\mu_{f,e} \\ &= \rho_{u,m}\,\rho_{e,m}\,\rho_{f,u}\,\rho_{f,e}\,\sigma_{f,u}\,\sigma_{f,e} \end{aligned} \tag{44}$$

The covariance between the returns on the chip business in Europe and the returns on the fax machine business in Europe (see Appendix equations (74), (75) and (85)) is...

$$\begin{aligned} Cov(r_{c,e} r_{f,e}) &= \mathbb{E}\Big[r_{c,e}\, r_{f,e}\Big] - \mathbb{E}\Big[r_{c,e}\Big]\mathbb{E}\Big[r_{f,e}\Big] \\ &= \mu_{c,e}\,\mu_{f,e} + \rho_{c,e}\,\rho_{f,e}\,\sigma_{c,e}\,\sigma_{f,e} - \mu_{c,e}\,\mu_{f,e} \\ &= \rho_{c,e}\,\rho_{f,e}\,\sigma_{c,e}\,\sigma_{f,e} \end{aligned} \tag{45}$$

The Solution To Our Hypothetical Problem

To solve the hypothetical problem above we will first need a column vector of portfolio weights. The portfolio weight vector **w** is...

$$\mathbf{w} = \begin{bmatrix} w_{c,u} \\ w_{f,u} \\ w_{c,e} \\ w_{f,e} \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.30 \\ 0.20 \\ 0.10 \end{bmatrix} \tag{46}$$

We will also need a column vector of expected returns and a column vector of expected return volatilities. The expected return vector **r** is...

$$\mathbf{r} = \begin{bmatrix} \mu_{c,u} \\ \mu_{f,u} \\ \mu_{c,e} \\ \mu_{f,e} \end{bmatrix} = \begin{bmatrix} 0.18 \\ 0.15 \\ 0.12 \\ 0.09 \end{bmatrix} \tag{47}$$

The expected volatility vector **v** is...

$$\mathbf{v} = \begin{bmatrix} \sigma_{c,u} \\ \sigma_{f,u} \\ \sigma_{c,e} \\ \sigma_{f,e} \end{bmatrix} = \begin{bmatrix} 0.30 \\ 0.25 \\ 0.20 \\ 0.15 \end{bmatrix} \tag{48}$$

To calculate portfolio volatility we will need the covariance matrix. The covariance matrix **C** is...

$$\mathbf{C} = \begin{bmatrix} Var(r_{c,u}) & Cov(r_{c,u}, r_{f,u}) & Cov(r_{c,u}, r_{c,e}) & Cov(r_{c,u}, r_{f,e}) \\ Cov(r_{f,u}, r_{c,u}) & Var(r_{f,u}) & Cov(r_{f,u}, r_{c,e}) & Cov(r_{f,u}, r_{f,e}) \\ Cov(r_{c,e}, r_{c,u}) & Cov(r_{c,e}, r_{f,u}) & Var(r_{c,e}) & Cov(r_{c,e}, r_{f,e}) \\ Cov(r_{f,e}, r_{c,u}) & Cov(r_{f,e}, r_{f,u}) & Cov(r_{f,e}, r_{c,e}) & Var(r_{f,e}) \end{bmatrix} = \begin{bmatrix} 0.0900 & 0.0446 & 0.0199 & 0.0112 \\ 0.0446 & 0.0625 & 0.0137 & 0.0077 \\ 0.0199 & 0.0137 & 0.0400 & 0.0144 \\ 0.0112 & 0.0077 & 0.0144 & 0.0225 \end{bmatrix} \tag{49}$$

Note: The above matrix and vectors make use of the tables in Our Hypothetical Problem above and the section on Product Returns, Variances and Covariances.

We now have enough information to answer the questions posed by our hypothetical problem above. The expected portfolio return (μ_p) is...

$$\mu_p = \mathbf{w}^T\mathbf{r} = 0.1500 \tag{50}$$

The weighted average portfolio volatility ($\hat{\sigma}_p$) is...

$$\hat{\sigma}_p = \mathbf{w}^T \mathbf{v} = 0.2500 \tag{51}$$

Expected portfolio volatility (σ_p), which reflects the benefits of diversification, is...

$$\sigma_p = \sqrt{\mathbf{w}^T \mathbf{C}\, \mathbf{w}} = \sqrt{0.0393} = 0.1983 \tag{52}$$

Conclusion: The weighted average portfolio volatility is 25%. Because of market and product diversification (and given that correlations are less than one) expected portfolio return volatility is approximately 20%, which is a 5% decrease over the simple weighted average volatility. Given a scenario where (1) there is market and product diversification and (2) relevant correlations are less than one, using a public company's volatility as a proxy for private company volatility may understate private company volatility when the private company is not diversified by market and/or by product.

To get some idea as to the sensitivity of the correlation assumptions lets assume that for the hypothetical problem above the factor correlation coefficients were equal, the portfolio weights were equal and the individual product volatilities were all 25%, which is the weighted average volatility. The following table shows total portfolio volatility assuming different levels of correlation...

Factor Correlation	Portfolio Volatility	Percent of Base Case *
1.00	25.00%	100%
0.90	22.09%	88%
0.80	19.60%	78%
0.70	17.55%	70%

* The base case assumes zero diversification benefit (i.e. correlations = 1.00)

To answer the question 'by how much is ABC Company's portfolio volatility reduced as the result of diversification as compared to a company with one product in one market' we can refer to the scenario table above. If the private company with one product in one market has an actual return volatility of 25% then ABC Company's portfolio volatility of 25% is a good proxy for the return volatilities of our private company. However, if correlations = 0.70 then ABC Company's portfolio volatility of 17.55% understates the return volatilities of our private company. In such cases where correlations are less than one Total Beta may yield a cost of capital that is a 'Lower Bound' to cost of capital for our private company.

Appendix

A. The first moment of the probability distribution of M_u as defined by Equation (2) is...

$$\begin{aligned} \mathbb{E}\Big[M_u\Big] &= \mathbb{E}\Big[\rho_{u,g}\, M_G + \sqrt{1-\rho_{u,g}^2}\, Z_u\Big] \\ &= \mathbb{E}\Big[\rho_{u,g}\, M_G\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{u,g}^2}\, Z_u\Big] \\ &= \rho_{u,g}\, \mathbb{E}\Big[M_G\Big] + \sqrt{1-\rho_{u,g}^2}\, \mathbb{E}\Big[Z_u\Big] \\ &= 0 \end{aligned} \tag{53}$$

Note that $\mathbb{E}[M_G] = 0$ per Equation (1) and $\mathbb{E}[Z_u] = 0$ per Equation (2).

B. The second moment of the probability distribution of M_u as defined by Equation (2) is...

$$\begin{aligned}
\mathbb{E}\Big[M_u^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{u,g}\,M_G + \sqrt{1-\rho_{u,g}^2}\,Z_u\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{u,g}^2\,M_G^2 + (1-\rho_{u,g}^2)\,Z_u^2 + 2\,\rho_{u,g}\sqrt{1-\rho_{u,g}^2}\,M_G\,Z_u\Big] \\
&= \mathbb{E}\Big[\rho_{u,g}^2\,M_G^2\Big] + \mathbb{E}\Big[(1-\rho_{u,g}^2)\,Z_u^2\Big] + \mathbb{E}\Big[2\,\rho_{u,g}\sqrt{1-\rho_{u,g}^2}\,M_G\,Z_u\Big] \\
&= \rho_{u,g}^2\,\mathbb{E}\Big[M_G^2\Big] + (1-\rho_{u,g}^2)\,\mathbb{E}\Big[Z_u^2\Big] + 2\,\rho_{u,g}\sqrt{1-\rho_{u,g}^2}\,\mathbb{E}\Big[M_G\,Z_u\Big] \\
&= \rho_{u,g}^2 + (1-\rho_{u,g}^2) \\
&= 1
\end{aligned} \tag{54}$$

Note that $\mathbb{E}[M_G^2] = 1$ per Equation (1), $\mathbb{E}[Z_u^2] = 1$ per Equation (2) and $\mathbb{E}[M_G\,Z_u] = 0$ because M_G and Z_u are independent by definition.

C. The first moment of the probability distribution of M_e as defined by Equation (6) is...

$$\begin{aligned}
\mathbb{E}\Big[M_e\Big] &= \mathbb{E}\Big[\rho_{e,g}\,M_G + \sqrt{1-\rho_{e,g}^2}\,Z_e\Big] \\
&= \mathbb{E}\Big[\rho_{e,g}\,M_G\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{e,g}^2}\,Z_e\Big] \\
&= \rho_{e,g}\,\mathbb{E}\Big[M_G\Big] + \sqrt{1-\rho_{e,g}^2}\,\mathbb{E}\Big[Z_e\Big] \\
&= 0
\end{aligned} \tag{55}$$

Note that $\mathbb{E}[M_G] = 0$ per Equation (1) and $\mathbb{E}[Z_e] = 0$ per Equation (6).

D. The second moment of the probability distribution of M_e as defined by Equation (6) is...

$$\begin{aligned}
\mathbb{E}\Big[M_e^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{e,g}\,M_G + \sqrt{1-\rho_{e,g}^2}\,Z_e\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{e,g}^2\,M_G^2 + (1-\rho_{e,g}^2)\,Z_e^2 + 2\,\rho_{e,g}\sqrt{1-\rho_{e,g}^2}\,M_G\,Z_e\Big] \\
&= \mathbb{E}\Big[\rho_{e,g}^2\,M_G^2\Big] + \mathbb{E}\Big[(1-\rho_{e,g}^2)\,Z_e^2\Big] + \mathbb{E}\Big[2\,\rho_{e,g}\sqrt{1-\rho_{e,g}^2}\,M_G\,Z_e\Big] \\
&= \rho_{e,g}^2\,\mathbb{E}\Big[M_G^2\Big] + (1-\rho_{e,g}^2)\,\mathbb{E}\Big[Z_e^2\Big] + 2\,\rho_{e,g}\sqrt{1-\rho_{e,g}^2}\,\mathbb{E}\Big[M_G\,Z_e\Big] \\
&= \rho_{e,g}^2 + (1-\rho_{e,g}^2) \\
&= 1
\end{aligned} \tag{56}$$

Note that $\mathbb{E}[M_G^2] = 1$ per Equation (1), $\mathbb{E}[Z_e^2] = 1$ per Equation (6) and $\mathbb{E}[M_G\,Z_e] = 0$ because of independence.

E. The expected value of the product of M_u and M_e is...

$$\begin{aligned}
\mathbb{E}\Big[M_u\,M_e\Big] &= \mathbb{E}\Big[\Big\{\rho_{u,m}\,M_G + \sqrt{1-\rho_{u,m}^2}\,Z_u\Big\}\Big\{\rho_{e,m}\,M_G + \sqrt{1-\rho_{e,m}^2}\,Z_e\Big\}\Big] \\
&= \mathbb{E}\Big[\rho_{u,m}\,\rho_{e,m}\,M_G^2 + \rho_{u,m}\,\sqrt{1-\rho_{e,m}^2}\,M_G\,Z_e + \rho_{e,m}\,\sqrt{1-\rho_{u,m}^2}\,M_G\,Z_u + \sqrt{1-\rho_{u,m}^2}\sqrt{1-\rho_{e,m}^2}\,Z_u\,Z_e\Big] \\
&= \mathbb{E}\Big[\rho_{u,m}\,\rho_{e,m}\,M_G^2\Big] + \mathbb{E}\Big[\rho_{u,m}\,\sqrt{1-\rho_{e,m}^2}\,M_G\,Z_e\Big] + \mathbb{E}\Big[\rho_{e,m}\,\sqrt{1-\rho_{u,m}^2}\,M_G\,Z_u\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{u,m}^2}\sqrt{1-\rho_{e,m}^2}\,Z_u\,Z_e\Big] \\
&= \rho_{u,m}\,\rho_{e,m}\,\mathbb{E}\Big[M_G^2\Big] + \rho_{u,m}\,\sqrt{1-\rho_{e,m}^2}\,\mathbb{E}\Big[M_G\,Z_e\Big] + \rho_{e,m}\,\sqrt{1-\rho_{u,m}^2}\,\mathbb{E}\Big[M_G\,Z_u\Big] + \sqrt{1-\rho_{u,m}^2}\sqrt{1-\rho_{e,m}^2}\,\mathbb{E}\Big[Z_u\,Z_e\Big] \\
&= \rho_{u,m}\,\rho_{e,m}
\end{aligned} \tag{57}$$

Note that $\mathbb{E}[M_G^2] = 1$ per Equation (1) and $\mathbb{E}[M_G\, Z_u] = \mathbb{E}[M_G\, Z_e] = \mathbb{E}[Z_u\, Z_e] = 0$ because of independence.

F. The expected value of C_u is...

$$
\begin{aligned}
\mathbb{E}\Big[C_u\Big] &= \mathbb{E}\Big[\rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, M_u\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big] \\
&= \rho_{c,u}\, \mathbb{E}\Big[M_u\Big] + \sqrt{1-\rho_{c,u}^2}\, \mathbb{E}\Big[Z_{c,u}\Big] \\
&= 0
\end{aligned} \tag{58}
$$

Note that $\mathbb{E}[M_u] = 0$ per Equation (53) and $\mathbb{E}[Z_{c,u}] = 0$ per Equation (12).

G. The expected value of C_u^2 is...

$$
\begin{aligned}
\mathbb{E}\Big[C_u^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}^2 M_u^2 + 2\,\rho_{c,u}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u} + (1-\rho_{c,u}^2)\, Z_{c,u}^2\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}^2 M_u^2\Big] + \mathbb{E}\Big[2\,\rho_{c,u}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u}\Big] + \mathbb{E}\Big[(1-\rho_{c,u}^2)\, Z_{c,u}^2\Big] \\
&= \rho_{c,u}^2\, \mathbb{E}\Big[M_u^2\Big] + 2\,\rho_{c,u}\sqrt{1-\rho_{c,u}^2}\, \mathbb{E}\Big[M_u\, Z_{c,u}\Big] + (1-\rho_{c,u}^2)\, \mathbb{E}\Big[Z_{c,u}^2\Big] \\
&= \rho_{c,u}^2 + (1-\rho_{c,u}^2) \\
&= 1
\end{aligned} \tag{59}
$$

Note that $\mathbb{E}[M_u^2] = 1$ per Equation (54), $\mathbb{E}[Z_{c,u}^2] = 1$ per Equation (12) and $\mathbb{E}[M_u\, Z_{c,u}] = 0$ because of independence.

H. The expected value of F_u is...

$$
\begin{aligned}
\mathbb{E}\Big[F_u\Big] &= \mathbb{E}\Big[\rho_{f,u}\, M_u + \sqrt{1-\rho_{f,u}^2}\, Z_{f,u}\Big] \\
&= \mathbb{E}\Big[\rho_{f,u}\, M_u\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{f,u}^2}\, Z_{f,u}\Big] \\
&= \rho_{f,u}\, \mathbb{E}\Big[M_u\Big] + \sqrt{1-\rho_{f,u}^2}\, \mathbb{E}\Big[Z_{f,u}\Big] \\
&= 0
\end{aligned} \tag{60}
$$

Note that $\mathbb{E}[M_u] = 0$ per Equation (53) and $\mathbb{E}[Z_{f,u}] = 0$ per Equation (13).

I. The expected value of F_u^2 is...

$$
\begin{aligned}
\mathbb{E}\Big[F_u^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{f,u}\, M_u + \sqrt{1-\rho_{f,u}^2}\, Z_{f,u}\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{f,u}^2 M_u^2 + 2\,\rho_{f,u}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{f,u} + (1-\rho_{f,u}^2)\, Z_{f,u}^2\Big] \\
&= \mathbb{E}\Big[\rho_{f,u}^2 M_u^2\Big] + \mathbb{E}\Big[2\,\rho_{f,u}\sqrt{1-\rho_{f,u}^2}\, M_u\, Z_{f,u}\Big] + \mathbb{E}\Big[(1-\rho_{f,u}^2)\, Z_{f,u}^2\Big] \\
&= \rho_{f,u}^2\, \mathbb{E}\Big[M_u^2\Big] + 2\,\rho_{f,u}\sqrt{1-\rho_{f,u}^2}\, \mathbb{E}\Big[M_u\, Z_{f,u}\Big] + (1-\rho_{f,u}^2)\, \mathbb{E}\Big[Z_{f,u}^2\Big] \\
&= \rho_{f,u}^2 + (1-\rho_{f,u}^2) \\
&= 1
\end{aligned} \tag{61}
$$

Note that $\mathbb{E}[M_u^2] = 1$ per Equation (54), $\mathbb{E}[Z_{fu}^2] = 1$ per Equation (13), and $\mathbb{E}[M_u\, Z_{f,u}] = 0$ because of independence.

J. The expected value of C_e is...

$$\begin{aligned}
\mathbb{E}\Big[C_e\Big] &= \mathbb{E}\Big[\rho_{c,e}\, M_e + \sqrt{1-\rho_{c,e}^2}\, Z_{c,e}\Big] \\
&= \mathbb{E}\Big[\rho_{c,e}\, M_e\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{c,e}^2}\, Z_{c,e}\Big] \\
&= \rho_{c,e}\, \mathbb{E}\Big[M_e\Big] + \sqrt{1-\rho_{c,e}^2}\, \mathbb{E}\Big[Z_{c,e}\Big] \\
&= 0
\end{aligned} \tag{62}$$

Note that $\mathbb{E}[M_e] = 0$ per Equation (55) and $\mathbb{E}[Z_{c,e}] = 0$ per Equation (20).

K. The expected value of C_e^2 is...

$$\begin{aligned}
\mathbb{E}\Big[C_e^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{c,e}\, M_e + \sqrt{1-\rho_{c,e}^2}\, Z_{c,u}\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{c,e}^2 M_e^2 + 2\,\rho_{c,e}\sqrt{1-\rho_{c,e}^2}\, M_e\, Z_{c,e} + (1-\rho_{c,e}^2)\, Z_{c,e}^2\Big] \\
&= \mathbb{E}\Big[\rho_{c,e}^2 M_e^2\Big] + \mathbb{E}\Big[2\,\rho_{c,e}\sqrt{1-\rho_{c,e}^2}\, M_u\, Z_{c,e}\Big] + \mathbb{E}\Big[(1-\rho_{c,e}^2)\, Z_{c,e}^2\Big] \\
&= \rho_{c,e}^2\, \mathbb{E}\Big[M_e^2\Big] + 2\,\rho_{c,e}\sqrt{1-\rho_{c,e}^2}\, \mathbb{E}\Big[M_e\, Z_{c,e}\Big] + (1-\rho_{c,e}^2)\, \mathbb{E}\Big[Z_{c,e}^2\Big] \\
&= \rho_{c,e}^2 + (1-\rho_{c,e}^2) \\
&= 1
\end{aligned} \tag{63}$$

Note that $\mathbb{E}[M_e^2] = 1$ per Equation (56), $\mathbb{E}[Z_{c,e}^2] = 1$ per Equation (20), and $\mathbb{E}[M_e\, Z_{c,e}] = 0$ because of independence.

L. The expected value of F_e is...

$$\begin{aligned}
\mathbb{E}\Big[F_e\Big] &= \mathbb{E}\Big[\rho_{f,e}\, M_e + \sqrt{1-\rho_{f,e}^2}\, Z_{f,e}\Big] \\
&= \mathbb{E}\Big[\rho_{f,e}\, M_e\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{f,e}^2}\, Z_{f,e}\Big] \\
&= \rho_{f,e}\, \mathbb{E}\Big[M_e\Big] + \sqrt{1-\rho_{f,e}^2}\, \mathbb{E}\Big[Z_{f,e}\Big] \\
&= 0
\end{aligned} \tag{64}$$

Note that $\mathbb{E}[M_e] = 0$ per Equation (55) and $\mathbb{E}[Z_{f,e}] = 0$ per Equation (21).

M. The expected value of F_e^2 is...

$$\begin{aligned}
\mathbb{E}\Big[F_e^2\Big] &= \mathbb{E}\Big[\Big\{\rho_{f,e}\, M_e + \sqrt{1-\rho_{f,e}^2}\, Z_{f,u}\Big\}^2\Big] \\
&= \mathbb{E}\Big[\rho_{f,e}^2 M_e^2 + 2\,\rho_{f,e}\sqrt{1-\rho_{c,e}^2}\, M_e\, Z_{f,e} + (1-\rho_{f,e}^2)\, Z_{f,e}^2\Big] \\
&= \mathbb{E}\Big[\rho_{f,e}^2 M_e^2\Big] + \mathbb{E}\Big[2\,\rho_{f,e}\sqrt{1-\rho_{f,e}^2}\, M_u\, Z_{f,e}\Big] + \mathbb{E}\Big[(1-\rho_{f,e}^2)\, Z_{f,u}^2\Big] \\
&= \rho_{f,e}^2\, \mathbb{E}\Big[M_e^2\Big] + 2\,\rho_{f,eu}\sqrt{1-\rho_{f,e}^2}\, \mathbb{E}\Big[M_e\, Z_{f,e}\Big] + (1-\rho_{f,e}^2)\, \mathbb{E}\Big[Z_{f,e}^2\Big] \\
&= \rho_{f,e}^2 + (1-\rho_{f,e}^2) \\
&= 1
\end{aligned} \tag{65}$$

Note that $\mathbb{E}[M_e^2] = 1$ per Equation (56), $\mathbb{E}[Z_{f,e}^2] = 1$ per Equation (21), and $\mathbb{E}[M_e\, Z_{f,e}] = 0$ because of independence.

N. The expected value of the product of C_u and F_u is...

$$\begin{aligned}
\mathbb{E}\Big[C_u\, F_u\Big] &= \mathbb{E}\Big[\Big\{\rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big\}\Big\{\rho_{f,u}\, M_u + \sqrt{1-\rho_{f,u}^2}\, Z_{f,u}\Big\}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{f,u} M_u^2 + \rho_{c,u}\sqrt{1-\rho_{f,u}^2}\, M_u\, Z_{f,u} + \rho_{f,u}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u} + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,u}^2}\, Z_{c,u}\, Z_{f,u}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{f,u} M_u^2\Big] + \mathbb{E}\Big[\rho_{c,u}\sqrt{1-\rho_{f,u}^2}\, M_u\, Z_{f,u}\Big] + \mathbb{E}\Big[\rho_{f,u}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u}\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,u}^2}\, Z_{c,u}\, Z_{f,u}\Big] \\
&= \rho_{c,u}\, \rho_{f,u}\, \mathbb{E}\Big[M_u^2\Big] + \rho_{c,u}\sqrt{1-\rho_{f,u}^2}\, \mathbb{E}\Big[M_u\, Z_{f,u}\Big] + \rho_{f,u}\sqrt{1-\rho_{c,u}^2}\, \mathbb{E}\Big[M_u\, Z_{c,u}\Big] + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,u}^2}\, \mathbb{E}\Big[Z_{c,u}\, Z_{f,u}\Big] \\
&= \rho_{c,u}\, \rho_{f,u} \qquad (66)
\end{aligned}$$

Note that $\mathbb{E}[M_u^2] = 1$ per Equation (54) and $\mathbb{E}[M_u\, Z_{f,u}] = \mathbb{E}[M_u\, Z_{c,u}] = \mathbb{E}[Z_{c,u}\, Z_{f,u}] = 0$ because of independence.

O. The expected value of the product of C_u and C_e is...

$$\begin{aligned}
\mathbb{E}\Big[C_u\, C_e\Big] &= \mathbb{E}\Big[\Big\{\rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big\}\Big\{\rho_{c,e}\, M_e + \sqrt{1-\rho_{c,e}^2}\, Z_{c,e}\Big\}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{c,e} M_u M_e + \rho_{c,u}\sqrt{1-\rho_{c,e}^2}\, M_u\, Z_{c,e} + \rho_{c,e}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u} + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{c,e}^2}\, Z_{c,u}\, Z_{c,e}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{c,e} M_u M_e\Big] + \mathbb{E}\Big[\rho_{c,u}\sqrt{1-\rho_{c,e}^2}\, M_u\, Z_{c,e}\Big] + \mathbb{E}\Big[\rho_{c,e}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u}\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{c,e}^2}\, Z_{c,u}\, Z_{c,e}\Big] \\
&= \rho_{c,u}\, \rho_{c,e}\, \mathbb{E}\Big[M_u M_e\Big] + \rho_{c,u}\sqrt{1-\rho_{c,e}^2}\, \mathbb{E}\Big[M_u\, Z_{c,e}\Big] + \rho_{c,e}\sqrt{1-\rho_{c,u}^2}\, \mathbb{E}\Big[M_u\, Z_{c,u}\Big] + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{c,e}^2}\, \mathbb{E}\Big[Z_{c,u}\, Z_{c,e}\Big] \\
&= \rho_{c,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m} \qquad (67)
\end{aligned}$$

Note that $\mathbb{E}[M_u\, M_e] = \rho_{u,m}\, \rho_{e,m}$ per Equation (57) and $\mathbb{E}[M_u\, Z_{c,e}] = \mathbb{E}[M_u\, Z_{c,u}] = \mathbb{E}[Z_{c,u}\, Z_{c,e}] = 0$ because of independence.

P. The expected value of the product of C_u and F_e is...

$$\begin{aligned}
\mathbb{E}\Big[C_u\, F_e\Big] &= \mathbb{E}\Big[\Big\{\rho_{c,u}\, M_u + \sqrt{1-\rho_{c,u}^2}\, Z_{c,u}\Big\}\Big\{\rho_{f,e}\, M_e + \sqrt{1-\rho_{f,e}^2}\, Z_{f,e}\Big\}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{f,e} M_u M_e + \rho_{c,u}\sqrt{1-\rho_{f,e}^2}\, M_u\, Z_{f,e} + \rho_{f,e}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u} + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,e}^2}\, Z_{c,u}\, Z_{f,e}\Big] \\
&= \mathbb{E}\Big[\rho_{c,u}\, \rho_{f,e} M_u M_e\Big] + \mathbb{E}\Big[\rho_{c,u}\sqrt{1-\rho_{f,e}^2}\, M_u\, Z_{f,e}\Big] + \mathbb{E}\Big[\rho_{f,e}\sqrt{1-\rho_{c,u}^2}\, M_u\, Z_{c,u}\Big] + \mathbb{E}\Big[\sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,e}^2}\, Z_{c,u}\, Z_{f,e}\Big] \\
&= \rho_{c,u}\, \rho_{f,e}\, \mathbb{E}\Big[M_u M_e\Big] + \rho_{c,u}\sqrt{1-\rho_{f,e}^2}\, \mathbb{E}\Big[M_u\, Z_{f,e}\Big] + \rho_{c,e}\sqrt{1-\rho_{c,u}^2}\, \mathbb{E}\Big[M_u\, Z_{c,u}\Big] + \sqrt{1-\rho_{c,u}^2}\sqrt{1-\rho_{f,e}^2}\, \mathbb{E}\Big[Z_{c,u}\, Z_{f,e}\Big] \\
&= \rho_{c,u}\, \rho_{f,e}\, \rho_{u,m}\, \rho_{e,m} \qquad (68)
\end{aligned}$$

Note that $\mathbb{E}[M_u\, M_e] = \rho_{u,m}\, \rho_{e,m}$ per Equation (57) and $\mathbb{E}[M_u\, Z_{f,e}] = \mathbb{E}[M_u\, Z_{c,u}] = \mathbb{E}[Z_{c,u}\, Z_{f,e}] = 0$ because of independence.

Q. The expected value of the product of F_u and C_e is...

$$
\begin{aligned}
\mathbb{E}\Big[F_u\,C_e\Big] &= \mathbb{E}\Bigg[\bigg\{\rho_{f,u}\,M_u + \sqrt{1-\rho_{f,u}^2}\,Z_{f,u}\bigg\}\bigg\{\rho_{c,e}\,M_e + \sqrt{1-\rho_{c,e}^2}\,Z_{c,e}\bigg\}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{f,u}\,\rho_{c,e}M_u M_e + \rho_{f,u}\,\sqrt{1-\rho_{c,e}^2}\,M_u\,Z_{c,e} + \rho_{c,e}\,\sqrt{1-\rho_{f,u}^2}\,M_e\,Z_{f,u} + \sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{c,e}^2}\,Z_{f,u}\,Z_{c,e}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{f,u}\,\rho_{c,e}M_u M_e\Bigg] + \mathbb{E}\Bigg[\rho_{f,u}\,\sqrt{1-\rho_{c,e}^2}\,M_u\,Z_{c,e}\Bigg] + \mathbb{E}\Bigg[\rho_{c,e}\,\sqrt{1-\rho_{f,u}^2}\,M_e\,Z_{f,u}\Bigg] + \mathbb{E}\Bigg[\sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{c,e}^2}\,Z_{f,u}\,Z_{c,e}\Bigg] \\
&= \rho_{f,u}\,\rho_{c,e}\,\mathbb{E}\Bigg[M_u M_e\Bigg] + \rho_{f,u}\,\sqrt{1-\rho_{c,e}^2}\,\mathbb{E}\Bigg[M_u\,Z_{c,e}\Bigg] + \rho_{c,e}\,\sqrt{1-\rho_{f,u}^2}\,\mathbb{E}\Bigg[M_e\,Z_{f,u}\Bigg] + \sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{c,e}^2}\,\mathbb{E}\Bigg[Z_{f,u}\,Z_{c,e}\Bigg] \\
&= \rho_{f,u}\,\rho_{c,e}\,\rho_{u,m}\,\rho_{e,m}
\end{aligned}
\tag{69}
$$

Note that $\mathbb{E}[M_u\,M_e] = \rho_{u,m}\,\rho_{e,m}$ per Equation (57) and $\mathbb{E}[M_u\,Z_{c,e}] = \mathbb{E}[M_e\,Z_{f,u}] = \mathbb{E}[Z_{f,u}\,Z_{c,e}] = 0$ because of independence.

R. The expected value of the product of F_u and F_e is...

$$
\begin{aligned}
\mathbb{E}\Big[F_u\,F_e\Big] &= \mathbb{E}\Bigg[\bigg\{\rho_{f,u}\,M_u + \sqrt{1-\rho_{f,u}^2}\,Z_{f,u}\bigg\}\bigg\{\rho_{f,e}\,M_e + \sqrt{1-\rho_{f,e}^2}\,Z_{f,e}\bigg\}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{f,u}\,\rho_{f,e}M_u M_e + \rho_{f,u}\,\sqrt{1-\rho_{f,e}^2}\,M_u\,Z_{f,e} + \rho_{f,e}\,\sqrt{1-\rho_{f,u}^2}\,M_e\,Z_{f,u} + \sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{f,e}^2}\,Z_{f,u}\,Z_{f,e}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{f,u}\,\rho_{f,e}M_u M_e\Bigg] + \mathbb{E}\Bigg[\rho_{f,u}\,\sqrt{1-\rho_{f,e}^2}\,M_u\,Z_{f,e}\Bigg] + \mathbb{E}\Bigg[\rho_{f,e}\,\sqrt{1-\rho_{f,u}^2}\,M_e\,Z_{f,u}\Bigg] + \mathbb{E}\Bigg[\sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{f,e}^2}\,Z_{f,u}\,Z_{f,e}\Bigg] \\
&= \rho_{f,u}\,\rho_{f,e}\,\mathbb{E}\Bigg[M_u M_e\Bigg] + \rho_{f,u}\,\sqrt{1-\rho_{f,e}^2}\,\mathbb{E}\Bigg[M_u\,Z_{f,e}\Bigg] + \rho_{f,e}\,\sqrt{1-\rho_{f,u}^2}\,\mathbb{E}\Bigg[M_e\,Z_{f,u}\Bigg] + \sqrt{1-\rho_{f,u}^2}\sqrt{1-\rho_{f,e}^2}\,\mathbb{E}\Bigg[Z_{f,u}\,Z_{f,e}\Bigg] \\
&= \rho_{f,u}\,\rho_{f,e}\,\rho_{u,m}\,\rho_{e,m}
\end{aligned}
\tag{70}
$$

Note that $\mathbb{E}[M_u\,M_e] = \rho_{u,m}\,\rho_{e,m}$ per Equation (57) and $\mathbb{E}[M_u\,Z_{f,e}] = \mathbb{E}[M_e\,Z_{f,u}] = \mathbb{E}[Z_{f,u}\,Z_{f,e}] = 0$ because of independence.

S. The expected value of the product of C_e and F_e is...

$$
\begin{aligned}
\mathbb{E}\Big[C_e\,F_e\Big] &= \mathbb{E}\Bigg[\bigg\{\rho_{c,e}\,M_e + \sqrt{1-\rho_{c,e}^2}\,Z_{c,e}\bigg\}\bigg\{\rho_{f,e}\,M_e + \sqrt{1-\rho_{f,e}^2}\,Z_{f,e}\bigg\}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{c,e}\,\rho_{f,e}M_e^2 + \rho_{c,u}\,\sqrt{1-\rho_{f,e}^2}\,M_e\,Z_{f,e} + \rho_{f,e}\,\sqrt{1-\rho_{c,e}^2}\,M_e\,Z_{c,e} + \sqrt{1-\rho_{c,e}^2}\sqrt{1-\rho_{f,e}^2}\,Z_{c,e}\,Z_{f,e}\Bigg] \\
&= \mathbb{E}\Bigg[\rho_{c,e}\,\rho_{f,e}M_e^2\Bigg] + \mathbb{E}\Bigg[\rho_{c,e}\,\sqrt{1-\rho_{f,e}^2}\,M_e\,Z_{f,e}\Bigg] + \mathbb{E}\Bigg[\rho_{f,e}\,\sqrt{1-\rho_{c,e}^2}\,M_e\,Z_{c,e}\Bigg] + \mathbb{E}\Bigg[\sqrt{1-\rho_{c,e}^2}\sqrt{1-\rho_{f,e}^2}\,Z_{c,e}\,Z_{f,e}\Bigg] \\
&= \rho_{c,e}\,\rho_{f,e}\,\mathbb{E}\Bigg[M_e^2\Bigg] + \rho_{c,e}\,\sqrt{1-\rho_{f,e}^2}\,\mathbb{E}\Bigg[M_e\,Z_{f,e}\Bigg] + \rho_{f,e}\,\sqrt{1-\rho_{c,e}^2}\,\mathbb{E}\Bigg[M_e\,Z_{c,e}\Bigg] + \sqrt{1-\rho_{c,e}^2}\sqrt{1-\rho_{f,e}^2}\,\mathbb{E}\Bigg[Z_{c,e}\,Z_{f,e}\Bigg] \\
&= \rho_{c,e}\,\rho_{f,e}
\end{aligned}
\tag{71}
$$

Note that $\mathbb{E}[M_e^2] = 1$ per Equation (57) and $\mathbb{E}[M_e\,Z_{f,u}] = \mathbb{E}[M_e\,Z_{c,e}] = \mathbb{E}[Z_{c,e}\,Z_{f,e}] = 0$ because of independence.

T. The expected value of the return on the chip business in The United States is...

$$
\begin{aligned}
\mathbb{E}\Bigg[r_{c,u}\Bigg] &= \mathbb{E}\Bigg[\mu_{c,u} + \sigma_{c,u}\,C_u\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{c,u}\Bigg] + \mathbb{E}\Bigg[\sigma_{c,u}\,C_u\Bigg] \\
&= \mu_{c,u} + \sigma_{c,u}\,\mathbb{E}\Bigg[C_u\Bigg] \\
&= \mu_{c,u}
\end{aligned}
\tag{72}
$$

Note that $\mathbb{E}[C_u] = 0$ per Equation (58).

U. The expected value of the return on the fax business in The United States is...

$$\begin{aligned}
\mathbb{E}\Big[r_{f,u}\Big] &= \mathbb{E}\Big[\mu_{f,u} + \sigma_{f,u}\, C_u\Big] \\
&= \mathbb{E}\Big[\mu_{f,u}\Big] + \mathbb{E}\Big[\sigma_{f,u}\, C_u\Big] \\
&= \mu_{f,u} + \sigma_{f,u}\, \mathbb{E}\Big[C_u\Big] \\
&= \mu_{f,u}
\end{aligned} \tag{73}$$

Note that $\mathbb{E}[C_u] = 0$ per Equation (58).

V. The expected value of the return on the chip business in Europe is...

$$\begin{aligned}
\mathbb{E}\Big[r_{c,e}\Big] &= \mathbb{E}\Big[\mu_{c,e} + \sigma_{c,e}\, C_e\Big] \\
&= \mathbb{E}\Big[\mu_{c,e}\Big] + \mathbb{E}\Big[\sigma_{c,e}\, C_e\Big] \\
&= \mu_{c,e} + \sigma_{c,u}\, \mathbb{E}\Big[C_e\Big] \\
&= \mu_{c,e}
\end{aligned} \tag{74}$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62).

W. The expected value of the return on the fax business in Europe is...

$$\begin{aligned}
\mathbb{E}\Big[r_{f,e}\Big] &= \mathbb{E}\Big[\mu_{f,e} + \sigma_{f,e}\, C_e\Big] \\
&= \mathbb{E}\Big[\mu_{f,e}\Big] + \mathbb{E}\Big[\sigma_{f,e}\, C_e\Big] \\
&= \mu_{f,e} + \sigma_{f,e}\, \mathbb{E}\Big[C_e\Big] \\
&= \mu_{f,e}
\end{aligned} \tag{75}$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62).

X. The expected value of the squared return on the chip business in The United States is...

$$\begin{aligned}
\mathbb{E}\Big[r_{c,u}^2\Big] &= \mathbb{E}\Big[\Big\{\mu_{c,u} + \sigma_{c,u}\, C_u\Big\}^2\Big] \\
&= \mathbb{E}\Big[\mu_{c,u}^2 + 2\,\mu_{c,u}\,\sigma_{c,u}\, C_u + \sigma_{c,u}^2\, C_u^2\Big] \\
&= \mathbb{E}\Big[\mu_{c,u}^2\Big] + \mathbb{E}\Big[2\,\mu_{c,u}\,\sigma_{c,u}\, C_u\Big] + \mathbb{E}\Big[\sigma_{c,u}^2\, C_u^2\Big] \\
&= \mu_{c,u}^2 + 2\,\mu_{c,u}\,\sigma_{c,u}\, \mathbb{E}\Big[C_u\Big] + \sigma_{c,u}^2\, \mathbb{E}\Big[C_u^2\Big] \\
&= \mu_{c,u}^2 + \sigma_{c,u}^2
\end{aligned} \tag{76}$$

Note that $\mathbb{E}[C_u] = 0$ per Equation (58) and $\mathbb{E}[C_u^2] = 1$ per Equation (59)

Y. The expected value of the squared return on the fax machine business in The United States is...

$$\begin{aligned}
\mathbb{E}\left[r_{f,u}^2\right] &= \mathbb{E}\left[\left\{\mu_{f,u} + \sigma_{f,u}\, C_u\right\}^2\right] \\
&= \mathbb{E}\left[\mu_{f,u}^2 + 2\,\mu_{f,u}\,\sigma_{f,u}\,C_u + \sigma_{f,u}^2\,C_u^2\right] \\
&= \mathbb{E}\left[\mu_{f,u}^2\right] + \mathbb{E}\left[2\,\mu_{f,u}\,\sigma_{f,u}\,C_u\right] + \mathbb{E}\left[\sigma_{f,u}^2\,C_u^2\right] \\
&= \mu_{f,u}^2 + 2\,\mu_{f,u}\,\sigma_{f,u}\,\mathbb{E}\left[C_u\right] + \sigma_{f,u}^2\,\mathbb{E}\left[C_u^2\right] \\
&= \mu_{f,u}^2 + \sigma_{f,u}^2
\end{aligned} \tag{77}$$

Note that $\mathbb{E}[C_u] = 0$ per Equation (58) and $\mathbb{E}[C_u^2] = 1$ per Equation (59)

Z. The expected value of the squared return on the chip business in Europe is...

$$\begin{aligned}
\mathbb{E}\left[r_{c,e}^2\right] &= \mathbb{E}\left[\left\{\mu_{c,e} + \sigma_{c,e}\, C_e\right\}^2\right] \\
&= \mathbb{E}\left[\mu_{c,e}^2 + 2\,\mu_{c,e}\,\sigma_{c,e}\,C_e + \sigma_{c,e}^2\,C_e^2\right] \\
&= \mathbb{E}\left[\mu_{c,e}^2\right] + \mathbb{E}\left[2\,\mu_{c,e}\,\sigma_{c,e}\,C_e\right] + \mathbb{E}\left[\sigma_{c,e}^2\,C_u^2\right] \\
&= \mu_{c,e}^2 + 2\,\mu_{c,e}\,\sigma_{c,e}\,\mathbb{E}\left[C_e\right] + \sigma_{c,e}^2\,\mathbb{E}\left[C_e^2\right] \\
&= \mu_{c,e}^2 + \sigma_{c,e}^2
\end{aligned} \tag{78}$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62) and $\mathbb{E}[C_e^2] = 1$ per Equation (63)

AA. The expected value of the squared return on the fax machine business in Europe is...

$$\begin{aligned}
\mathbb{E}\left[r_{f,e}^2\right] &= \mathbb{E}\left[\left\{\mu_{f,e} + \sigma_{f,e}\, C_e\right\}^2\right] \\
&= \mathbb{E}\left[\mu_{f,e}^2 + 2\,\mu_{f,e}\,\sigma_{f,e}\,C_e + \sigma_{f,e}^2\,C_e^2\right] \\
&= \mathbb{E}\left[\mu_{f,e}^2\right] + \mathbb{E}\left[2\,\mu_{f,e}\,\sigma_{f,e}\,C_e\right] + \mathbb{E}\left[\sigma_{f,e}^2\,C_e^2\right] \\
&= \mu_{f,e}^2 + 2\,\mu_{f,e}\,\sigma_{f,e}\,\mathbb{E}\left[C_e\right] + \sigma_{f,e}^2\,\mathbb{E}\left[C_e^2\right] \\
&= \mu_{f,e}^2 + \sigma_{f,e}^2
\end{aligned} \tag{79}$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62) and $\mathbb{E}[C_e^2] = 1$ per Equation (63)

BB. The expected value of the product of the return on the chip business in The United States and the return on the fax machine business in The United States is...

$$\begin{aligned}
\mathbb{E}\left[r_{c,u}\,r_{f,u}\right] &= \mathbb{E}\left[\left\{\mu_{c,u} + \sigma_{c,u}\, C_u\right\}\left\{\mu_{f,u} + \sigma_{f,u}\, F_u\right\}\right] \\
&= \mathbb{E}\left[\mu_{c,u}\,\mu_{f,u} + \mu_{c,u}\,\sigma_{f,u}\,F_u + \mu_{f,u}\,\sigma_{c,u}\,C_u + \sigma_{c,u}\,\sigma_{f,u}\,C_u\,F_u\right] \\
&= \mathbb{E}\left[\mu_{c,u}\,\mu_{f,u}\right] + \mathbb{E}\left[\mu_{c,u}\,\sigma_{f,u}\,F_u\right] + \mathbb{E}\left[\mu_{f,u}\,\sigma_{c,u}\,C_u\right] + \mathbb{E}\left[\sigma_{c,u}\,\sigma_{f,u}\,C_u\,F_u\right] \\
&= \mu_{c,u}\,\mu_{f,u} + \mu_{c,u}\,\sigma_{f,u}\,\mathbb{E}\left[F_u\right] + \mu_{f,u}\,\sigma_{c,u}\,\mathbb{E}\left[C_u\right] + \sigma_{c,u}\,\sigma_{f,u}\,\mathbb{E}\left[C_u\,F_u\right] \\
&= \mu_{c,u}\,\mu_{f,u} + \sigma_{c,u}\,\sigma_{f,u}\left\{\rho_{c,u}\,\rho_{f,u}\right\}
\end{aligned} \tag{80}$$

Note that $\mathbb{E}[F_u] = 0$ per Equation (60), $\mathbb{E}[C_u] = 0$ per Equation (58) and $\mathbb{E}[C_u\, F_u] = \rho_{c,u}\, \rho_{f,u}$ per Equation (66).

CC. The expected value of the product of the return on the chip business in The United States and the return on the chip business in Europe is...

$$
\begin{aligned}
\mathbb{E}\Bigg[r_{c,u}\, r_{c,e}\Bigg] &= \mathbb{E}\Bigg[\Bigg\{\mu_{c,u} + \sigma_{c,u}\, C_u\Bigg\}\Bigg\{\mu_{c,e} + \sigma_{c,e}\, C_e\Bigg\}\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{c,u}\, \mu_{c,e} + \mu_{c,u}\, \sigma_{c,e}\, C_e + \mu_{c,e}\, \sigma_{c,u}\, C_u + \sigma_{c,u}\, \sigma_{c,e}\, C_u\, C_e\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{c,u}\, \mu_{c,e}\Bigg] + \mathbb{E}\Bigg[\mu_{c,u}\, \sigma_{c,e}\, C_e\Bigg] + \mathbb{E}\Bigg[\mu_{c,e}\, \sigma_{c,u}\, C_u\Bigg] + \mathbb{E}\Bigg[\sigma_{c,u}\, \sigma_{c,e}\, C_u\, C_e\Bigg] \\
&= \mu_{c,u}\, \mu_{c,e} + \mu_{c,u}\, \sigma_{c,e}\, \mathbb{E}\Bigg[C_e\Bigg] + \mu_{c,e}\, \sigma_{c,u}\, \mathbb{E}\Bigg[C_u\Bigg] + \sigma_{c,u}\, \sigma_{c,e}\, \mathbb{E}\Bigg[C_u\, C_e\Bigg] \\
&= \mu_{c,u}\, \mu_{c,e} + \sigma_{c,u}\, \sigma_{c,e} \Bigg\{\rho_{c,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m}\Bigg\} \qquad (81)
\end{aligned}
$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62), $\mathbb{E}[C_u] = 0$ per Equation (58) and $\mathbb{E}[C_u\, C_e] = \rho_{c,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m}$ per Equation (67).

DD. The expected value of the product of the return on the chip business in The United States and the return on the fax machine business in Europe is...

$$
\begin{aligned}
\mathbb{E}\Bigg[r_{c,u}\, r_{f,e}\Bigg] &= \mathbb{E}\Bigg[\Bigg\{\mu_{c,u} + \sigma_{c,u}\, C_u\Bigg\}\Bigg\{\mu_{f,e} + \sigma_{f,e}\, C_e\Bigg\}\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{c,u}\, \mu_{f,e} + \mu_{c,u}\, \sigma_{f,e}\, C_e + \mu_{f,e}\, \sigma_{c,u}\, C_u + \sigma_{c,u}\, \sigma_{f,e}\, C_u\, C_e\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{c,u}\, \mu_{f,e}\Bigg] + \mathbb{E}\Bigg[\mu_{c,u}\, \sigma_{f,e}\, C_e\Bigg] + \mathbb{E}\Bigg[\mu_{f,e}\, \sigma_{c,u}\, C_u\Bigg] + \mathbb{E}\Bigg[\sigma_{c,u}\, \sigma_{f,e}\, C_u\, C_e\Bigg] \\
&= \mu_{c,u}\, \mu_{f,e} + \mu_{c,u}\, \sigma_{f,e}\, \mathbb{E}\Bigg[C_e\Bigg] + \mu_{f,e}\, \sigma_{c,u}\, \mathbb{E}\Bigg[C_u\Bigg] + \sigma_{c,u}\, \sigma_{f,e}\, \mathbb{E}\Bigg[C_u\, C_e\Bigg] \\
&= \mu_{c,u}\, \mu_{f,e} + \sigma_{c,u}\, \sigma_{f,e} \Bigg\{\rho_{c,u}\, \rho_{f,e}\, \rho_{u,m}\, \rho_{e,m}\Bigg\} \qquad (82)
\end{aligned}
$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62), $\mathbb{E}[C_u] = 0$ per Equation (58) and $\mathbb{E}[C_u\, C_e] = \rho_{c,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m}$ per Equation (67).

EE. The expected value of the product of the return on the fax machine business in The United States and the return on the chip business in Europe is...

$$
\begin{aligned}
\mathbb{E}\Bigg[r_{f,u}\, r_{c,e}\Bigg] &= \mathbb{E}\Bigg[\Bigg\{\mu_{f,u} + \sigma_{f,u}\, F_u\Bigg\}\Bigg\{\mu_{c,e} + \sigma_{c,e}\, C_e\Bigg\} \\
&= \mathbb{E}\Bigg[\mu_{f,u}\, \mu_{c,e} + \mu_{f,u}\, \sigma_{c,e}\, C_e + \mu_{c,e}\, \sigma_{f,u}\, F_u + \sigma_{f,u}\, \sigma_{c,e}\, F_u\, C_e\Bigg] \\
&= \mathbb{E}\Bigg[\mu_{f,u}\, \mu_{c,e}\Bigg] + \mathbb{E}\Bigg[\mu_{f,u}\, \sigma_{c,e}\, C_e\Bigg] + \mathbb{E}\Bigg[\mu_{c,e}\, \sigma_{f,u}\, F_u\Bigg] + \mathbb{E}\Bigg[\sigma_{f,u}\, \sigma_{c,e}\, F_u\, C_e\Bigg] \\
&= \mu_{f,u}\, \mu_{c,e} + \mu_{f,u}\, \sigma_{c,e}\, \mathbb{E}\Bigg[C_e\Bigg] + \mu_{c,e}\, \sigma_{f,u}\, \mathbb{E}\Bigg[F_u\Bigg] + \sigma_{f,u}\, \sigma_{c,e}\, \mathbb{E}\Bigg[F_u\, C_e\Bigg] \\
&= \mu_{f,u}\, \mu_{c,e} + \sigma_{f,u}\, \sigma_{c,e} \Bigg\{\rho_{f,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m}\Bigg\} \qquad (83)
\end{aligned}
$$

Note that $\mathbb{E}[C_e] = 0$ per Equation (62), $\mathbb{E}\Bigg[F_u\Bigg] = 0$ per Equation (60) and $\mathbb{E}[F_u\, C_e] = \rho_{f,u}\, \rho_{c,e}\, \rho_{u,m}\, \rho_{e,m}$ per Equation (69).

FF. The expected value of the product of the return on the fax machine business in The United States and the return on the fax business in Europe is...

$$\begin{aligned}
\mathbb{E}\bigg[r_{f,u}\,r_{f,e}\bigg] &= \mathbb{E}\bigg[\bigg\{\mu_{f,u}+\sigma_{f,u}\,F_u\bigg\}\bigg\{\mu_{f,e}+\sigma_{f,e}\,F_e\bigg\}\bigg] \\
&= \mathbb{E}\bigg[\mu_{f,u}\,\mu_{f,e}+\mu_{f,u}\,\sigma_{f,e}\,F_e+\mu_{f,e}\,\sigma_{f,u}\,F_u+\sigma_{f,u}\,\sigma_{f,e}\,F_u\,F_e\bigg] \\
&= \mathbb{E}\bigg[\mu_{f,u}\,\mu_{f,e}\bigg]+\mathbb{E}\bigg[\mu_{f,u}\,\sigma_{f,e}\,F_e\bigg]+\mathbb{E}\bigg[\mu_{f,e}\,\sigma_{f,u}\,F_u\bigg]+\mathbb{E}\bigg[\sigma_{f,u}\,\sigma_{f,e}\,F_u\,F_e\bigg] \\
&= \mu_{f,u}\,\mu_{f,e}+\mu_{f,u}\,\sigma_{f,e}\,\mathbb{E}\bigg[F_e\bigg]+\mu_{f,e}\,\sigma_{f,u}\,\mathbb{E}\bigg[F_u\bigg]+\sigma_{f,u}\,\sigma_{f,e}\,\mathbb{E}\bigg[F_u\,F_e\bigg] \\
&= \mu_{f,u}\,\mu_{f,e}+\sigma_{f,u}\,\sigma_{f,e}\bigg\{\rho_{f,u}\,\rho_{f,e}\,\rho_{u,m}\,\rho_{e,m}\bigg\} \qquad (84)
\end{aligned}$$

Note that $\mathbb{E}[F_e]=0$ per Equation (64), $\mathbb{E}[F_u]=0$ per Equation (60) and $\mathbb{E}[F_u\,F_e]=\rho_{f,u}\,\rho_{f,e}\,\rho_{u,m}\,\rho_{e,m}$ per Equation (70).

GG. The expected value of the product of the return on the chip business in Europe and the return on the fax business in Europe is...

$$\begin{aligned}
\mathbb{E}\bigg[r_{c,e}\,r_{f,e}\bigg] &= \mathbb{E}\bigg[\bigg\{\mu_{c,e}+\sigma_{c,e}\,C_e\bigg\}\bigg\{\mu_{f,e}+\sigma_{f,e}\,F_e\bigg\}\bigg] \\
&= \mathbb{E}\bigg[\mu_{c,e}\,\mu_{f,e}+\mu_{c,e}\,\sigma_{f,e}\,F_e+\mu_{f,e}\,\sigma_{c,e}\,C_e+\sigma_{c,e}\,\sigma_{f,e}\,C_e\,F_e\bigg] \\
&= \mathbb{E}\bigg[\mu_{c,e}\,\mu_{f,e}\bigg]+\mathbb{E}\bigg[\mu_{c,e}\,\sigma_{f,e}\,F_e\bigg]+\mathbb{E}\bigg[\mu_{f,e}\,\sigma_{c,e}\,C_e\bigg]+\mathbb{E}\bigg[\sigma_{c,e}\,\sigma_{f,e}\,C_e\,F_e\bigg] \\
&= \mu_{c,e}\,\mu_{f,e}+\mu_{c,e}\,\sigma_{f,e}\,\mathbb{E}\bigg[F_e\bigg]+\mu_{f,e}\,\sigma_{c,e}\,\mathbb{E}\bigg[C_e\bigg]+\sigma_{c,e}\,\sigma_{f,e}\,\mathbb{E}\bigg[C_e\,F_e\bigg] \\
&= \mu_{c,e}\,\mu_{f,e}+\sigma_{c,e}\,\sigma_{f,e}\bigg\{\rho_{c,e}\,\rho_{f,e}\bigg\} \qquad (85)
\end{aligned}$$

Note that $\mathbb{E}[F_e]=0$ per Equation (64), $\mathbb{E}[C_e]=0$ per Equation (62) and $\mathbb{E}[C_e\,F_e]=\rho_{c,e}\,\rho_{f,e}$ per Equation (71).